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A finite volume model for coupling surface and subsurface flows

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Abstract

In this paper, a finite volume model using approximate Riemann solver is presented for solving the extended shallow water equations (ESWEs) which are suitable for both surface and subsurface flows. The surface gradient method is employed to balance the flux gradient and source terms. Firstly the model is verified against the analytical solutions for 1 Dimension dam break problem and the steady subsurface flow. Then we use the model to study the tidal flow across an embankment, and the results agree well with the experimental data. The numerical results show that the model is capable of simulating coupled surface and subsurface flows simultaneously without changing equations or numerical schemes. For simplicity, all the simulations are based on 1 Dimension, while the model can be easily extended to high dimension.

Keywords: Extended shallow water equations; finite volume method; approximate Riemann solver.

1. Introduction

The interaction between surface and subsurface flows has been studied by means of experiment and numerical simulation for a long time. Numerous integrated surface and subsurface flow models have been developed, which could be distinguished by the type of equations and the numerical schemes as well as the type of coupling. For surface water flow, two dimensional (2D) shallow water equations or one dimensional (1D) Saint-Venant equations are commonly used, while Darcy law or Richards equation is often used for

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subsurface flow [1]. Instead of Darcy law, some researchers [2-5] used Navier–Stokes (N-S) type model equations to study wave interaction with porous structures. In these models the effect of porous media is considered by adding an inertial force term and a nonlinear frictional force (drag force) term which is caused by the fixed solid skeleton, therefore a wide range of porous flows from laminar, transitional to fully turbulent flows could be modelled [5].

The coupling methods can be divided as external (loose) coupling and internal (tight) coupling. The difference between the two types is whether simulating surface and subsurface flows at the same time level or not [1]. Using the depth integrated shallow water equations for surface water flow and the extended Darcy's law for unconfined subsurface flow in isotropic and homogeneous media, Yuan et al. [6,7] proposed an implicit ADI technique to link the flows simultaneously in the same time step while Liang et al. [8] obtained an internal coupling method by using an explicit MacCormack scheme. Based on the work of Yuan et al. [6], Jun Kong [9] also presented a internal coupling scheme using an unstructured finite volume/finite difference method.

In this paper, extended shallow water equations were obtained on the basis of shallow water equations (SWEs) and the N-S type model equations in Section 2. In Section 3, numerical scheme was presented. In Section 4, three tests were given and the results were compared with the analytical solutions and the experimental data, respectively. Finally, the conclusions are summarized.

2. Governing Equation

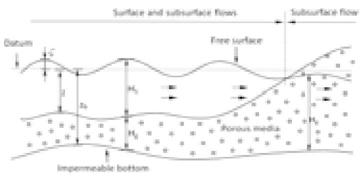


Figure 1 Sketch map of surface and subsurface flows

The 2D shallow water equations with the same form in Yuan et.al [7] are used for surface flow. For subsurface flow, the N-S type model equations [2-5] for porous flows are adopted. Herein by depth integration, the 3D N-S type equations can be converted into a similar form to that of the SWEs:

$$\frac{\partial H_g}{\partial t}+\frac{\partial\left(H_g u_g / n_e\right)}{\partial x}+\frac{\partial\left(H_g v_g / n_e\right)}{\partial y}=0 \qquad (1)$$

$$\frac{1}{n_e}\frac{\partial H_g u_g}{\partial t}+\frac{\partial\left(H_g u_g^{\,2} / n_e^{\,2}\right)}{\partial x}+\frac{\partial\left(H_g u_g v_g / n_e^{\,2}\right)}{\partial y}=-gH_g\frac{\partial\zeta}{\partial x}-F_{Dx} \qquad (2)$$

$$\frac{1}{n_e}\frac{\partial H_g v_g}{\partial t}+\frac{\partial\left(H_g u_g v_g / n_e^{\,2}\right)}{\partial x}+\frac{\partial\left(H_g v_g^{\,2} / n_e^{\,2}\right)}{\partial y}=-gH_g\frac{\partial\zeta}{\partial y}-F_{Dy} \qquad (3)$$

where, H_g is the total depth of the subsurface water, n_e is the porosity, u_g and v_g are depth-averaged velocity components in x and y directions respectively, F_{Dx} and F_{Dy} are drag force representing the effect of the fixed solid skeleton [2-5].

For the low Reynolds number flow in which the square law term can be ignored, if the permeability coefficient *K* is known, the drag force can be written as:

$$F_{Dx} = p_g g/K \, , \; F_{Dy} = q_g g/K \tag{4}$$

In this study, since the velocity of the subsurface water is small in the tests, the drag force is calculated by Eq. (4). If the unsteady term and the convection term are neglected, the momentum equations for subsurface flow can be reduced to the Darcy law, that is:

$$p_g = -KH_g \frac{\partial \zeta}{\partial x} \, , \; q_g = -KH_g \frac{\partial \zeta}{\partial y} \tag{5}$$

In the surface and subsurface flow region (Fig. 1) where the velocity of the subsurface water is assumed to be small, the subsurface flow is calculated explicitly by employing the Darcy law. Taking x direction for consideration, adding Eq. (4) to the momentum equation of the surface flow gives

$$\frac{\partial H_s u}{\partial t} + \frac{\partial \left(H_s u^2 \right)}{\partial x} + \frac{\partial \left(H_s uv \right)}{\partial y} = -g\left(H_T \right) \frac{\partial \zeta}{\partial x} - \tau_{bx} / \rho - F_{Dx} \tag{6}$$

in which $H_T = H_s + H_g$ is the total depth from the free surface to the impermeable bottom, τ_{bx} is the bed stress, $F_{Dx} = p_g g / K$. Comparing the governing equations for both surface flow and subsurface flow, a new set of equations can be obtained to describe the different flow schemes as follows:

$$\frac{\partial H}{\partial t} + \frac{\partial \left(Hu / n_e \right)}{\partial x} + \frac{\partial \left(Hv / n_e \right)}{\partial y} = 0 \tag{7}$$

$$\frac{\partial Hu}{\partial t} + \frac{\partial \left(Hu^2 / n_e \right)}{\partial x} + \frac{\partial \left(Huv / n_e \right)}{\partial y} = -gH_T \frac{\partial \zeta}{\partial x} n_e - \frac{\tau_{bx}}{\rho} \gamma n_e - F_{Dx} n_e \tag{8}$$

$$\frac{\partial Hv}{\partial t} + \frac{\partial \left(Huv / n_e \right)}{\partial x} + \frac{\partial \left(Hv^2 / n_e \right)}{\partial y} = -gH_T \frac{\partial \zeta}{\partial y} n_e - \frac{\tau_{by}}{\rho} \gamma n_e - F_{Dy} n_e \tag{9}$$

where $\gamma = \begin{cases} 1 & n_e = 0,1 \quad \text{for surface flow} \\ 0 & 0 \prec n_e \prec 1 \quad \text{for subsurface flow} \end{cases}$, H , u , v are water depth and velocity components for either surface flow or subsurface flow depending on the value of n_e.

3. Numerical Scheme

Approximate Riemann solver together with the surface gradient method is used to solve the extended shallow water. Roe's approximate Riemann solver [10] is applied to calculate numerical flux at each cell edge for surface flow. For subsurface flow, it was noticed that the Roe's approximate Riemann solver did not provide satisfactory results, so a simple arithmetic mean form of the numerical flux was used.

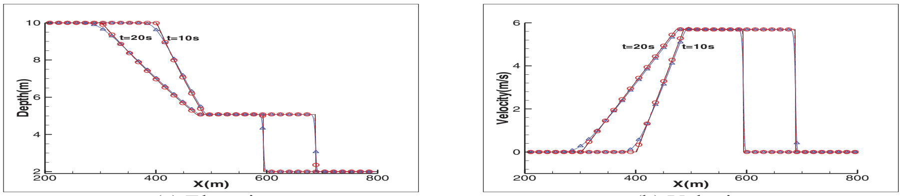
The surface gradient method (SGM) proposed by Zhou et al. [11] is used to balance bed slope source terms in the SWEs in this study. The virtue of the SGM is that it is as simple as the conventional method for homogeneous equations. The key of the SGM is to calculate the numerical flux using water level instead of the reconstructed depth itself. It should be noted that in the SGM, the bed slope source term should be discretized with a centered scheme to satisfy the exact C-property [11].

Boundary condition based on the theory of characteristics can be applied if the water level or velocity is known. Within the computational domain, moving boundary exists at the interface of surface and subsurface water. The total flux is linked at the interface to insure mess conservation.

4. Test Cases and results

4.1. One dimensional dam-break on a wet bed

Dam-break on a flat wet bed is chosen to test the ability of the numerical scheme to simulate the front propagation. In the test, the length of the channel is 1000 m with an initial break in the middle, and the initial depth on the left and right sides are 10 m and 2 m respectively. Wall boundary is set at the end of the channel. The space step is set as 1 m, with a time step of 0.01 s. It can be seen from Fig. 2 that the predicted elevation and velocity agree well with the analytical solution. No matter which order the reconstruction is, the front predicted are nearly the same as the exact front.



(a) Elevation (b) Velocity

Analytical Numerical Cons Numerical Superbee

Figure 2 Comparison of analytical H and u with predicted H and u

4.2. Steady subsurface flow

For unconfined aquifer, a simple test is steady subsurface flow through a vertical dam (Fig. 3). For a 1D problem with initial depth of H_1 and H_2 on two sides of the dam, ignoring source terms, the analytical groundwater level is a parabolic curve. In the test, the water levels are 10 m and 2 m respectively on the two sides of the dam with a length of 50 m, and the upstream and the downstream are surface water without porous medium below. The hydraulic conductivity is set as 0.01 m/s and 0.05 m/s, in two cases respectively. The cell length is 1 m and $\Delta t = 0.01\, s$.

As shown in Fig. 4, the free surface is in good agreement with the analytical solution regardless of the porosity and the hydraulic conductivity. The hydraulic conductivity just has effect on the flux as well as the time to be stable, but does not affect the water level.

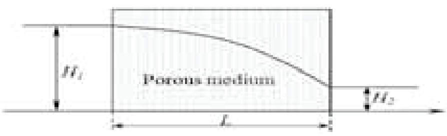


Figure 3 Flow through a porous vertical dam

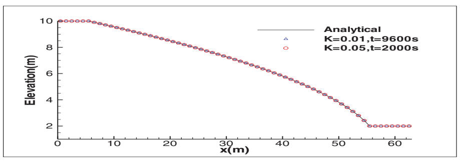


Figure 4 Predicted elevation

4.3. Tide across an experimental lagoon

An idealized experiment was carried out by Ebranhimi et al. [12] to study the influence of the tidal flow on a lagoon via a porous embankment. The experimental data were used to verify the capacity of the coupling model for simulating surface and subsurface flows simultaneously. The experiment setting, initial and boundary condition are the same as those used in [7-9]. In this model, the Manning's roughness coefficient is set to 0.011. Cell size is set as 2 cm with a time step of 0.012s.

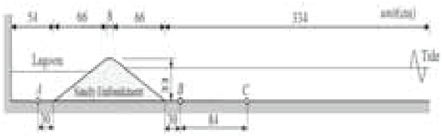


Figure 5 Side view and dimensions of experimental tidal lagoon

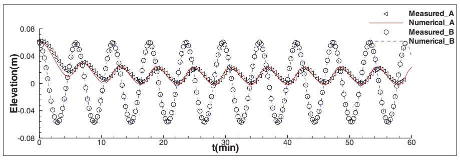


Figure 6 Elevation at points A and B

It can be seen from Fig. 6 that the predicted elevation at point *A* and *B* on the two sides of the embankment agree well with the experimental data. However, the water level predicted in the beginning two cycles is slightly lower than the recorded level at point *A*.

5. Conclusions

Surface and subsurface flows are coupled by solving the extended shallow water equations using a finite volume method with approximate Riemann solver. The ESWEs are obtained on the basis of the depth-integrated shallow water equations for surface flow and the N-S model type equations for subsurface flow. The model were verified against the analytical solutions for a 1D dam break problem and the steady subsurface flow, as well as the laboratory data.

The model is able to simulate different flow regimes such as steady and unsteady, subcritical and supercritical flow. The test of the 1D dam break demonstrates the capability of shock capturing of the model. And the steady subsurface flow and the tide simulation across the experimental lagoon show that the model is capable of simulating the water table fluctuations for both vertical and sloping beaches. Using the ESWEs, the solutions are easily implemented in a single structure. High Reynolds number flow for subsurface flow could be considered in the future work by adding the quadratic term in the drag force terms.

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